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A Review on Policies and Supply Chain Relationships under Inventory Transshipment

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Abstract

In distribution systems, excess stocks can be shared across different locations to fulfill unsatisfied demand. This raises many operations problems including inventory policies, transportation, partner selection and contracts. This paper reviews the up-to-date literature in inventory transshipment by considering different supply chain structures - centralized or decentralized, and whether the transshipment involves physical movement of the stocks, or is virtual. A number of future directions are provided in stimulating relevant studies.

Keywords: Transshipment, Inventory, Supply Chain, Centralization, Decentralization, Competition, Cooperation.
AMS Subject classifications: 90, 91

1. Introduction

Globalized supply chain involving complex business networks entails efficient policies for daily operations, and appropriate contracts in managing partner relationships. These issues are highlighted in the distribution systems where 1) the systems are geographically extended with multiple locations, 2) some common product is demanded at each location, and 3) customer service and responsiveness are satisfied via make-to-stock inventories. In managing systems of this nature, it is not trivial to consider the possibility that different locations may share their inventories — in particular, whether stocks can be transshiped (or customers be directed) from one location to another where demand takes place (or product is in stock). The legitimacy of this choice affects many relevant decisions including replenishing policy, transportation, partner selection, promotions and pricing. These issues are also subject to the change of time, supply chain structure, technology, and customer behaviour. Inventory transshipment has thus been long studied in operations literature.

This paper reviews the body of literature concerning transshipment of inventories between different locations in capturing upfront demand[1]. Specifically, we take into account both *physical* and *virtual* transshipment. Both happen when one location does not hold the product a customer needs while another location does. For *physical* transshipment, the product itself will be sent from the second location to the first one where demand takes place. The entire process is hence "seller-driven." On the other hand, *virtual* transshipment applies when the customer is informed (or as a result of searching for available sellers himself) and directed the second location for purchase. This process is hence "customer-driven" (note that certain demand might be lost due to some customers not willing to search or travel).

The general model framework used in the literature of transshipment is illustrated in Figure 1. It usually involves a manufacturer/supplier (or a source of order, if the entity itself is not the target of study) and a network of retailers (or warehouses/locations). In operations literature, links between firms at different stages (resp. the same stage) of a supply chain are referred to as vertical (resp. horizontal) relationships. Therefore, vertical relationships are relevant to firms with complementary skills and resources — in this context, between the upstream manufacturer/supplier and the downstream retailers. And horizontal relationships involve firms in the same echelon with similar/paralelle functions — in our model between or among the retailers. There are a few kinds of horizontal relationships among the retailers. *Centralized* models investigate the scenarios when all retailers are owned by one business entity. Otherwise if each retailer acts on it own, the system is *decentralized* and the retailers can behave *competitively* or *cooperatively* on operational decision makings. In Table 1, we classify transshipment papers according to the horizontal relationships among the retailers, and also whether a paper considers vertical relationships between the manufacturer and the retailers (e.g, manufacturer's pricing/rationing power upon the retailers, impact of transshipment on the manufacturer, etc.).

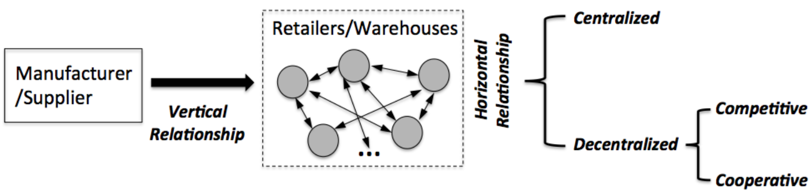

Figure 1: General Framework of Inventory Transshipment Models

[1] This does not include redistribution or preventive transshipment prior to demand arrival. Interested readers may refer to Paterson et al. (2011) for these lines of research.

2. Centralized Models

The stream of research on transshipment arises from optimizing distribution networks in which all retailers belong to the same entity. Relevant decisions, e.g., ordering and transshipment, are made by one central decision maker in maximizing the total profit of all retailers. In this section, we will review papers in this segment. These problems vary from periodic to continuous review models, retailers homogeneous to heterogeneous, and transshipment physical or virtual.

2.1. Physical transshipment in centralized models

We first summarize centralized models involving physical movement of the inventories. Relevant inventory systems can mainly be divided into periodic review systems and continuous review systems.

For periodic review systems, Krishnan and Rao (1965) consider a periodic model with two or more locations. At the beginning of each period, each location replenishes inventory according to its base-stock policy. Excess inventories will be transhipped across locations to satisfy unfulfilled demand. For locations with identical cost structure[2], the paper derives optimal base-stock policies that minimize the total of holding, shortage and transportation costs of all retailers.

For more general scenarios with heterogeneous cost structures, Tagaras (1989) considers two non-identical locations and illustrates the impact of pooling on service levels. It also discusses the optimal base-stock policy given a target fill-rate. Robinson (1990) analyzes optimal base-stock policies with two general locations (identical or non-identical) and n identical locations. For n non-identical locations it provides a heuristic solution, the savings of which can be significant especially for large n and small transshipment costs. For n general locations, Herer et al. (2006) also investigate determining optimal base-stock levels through Infinitesimal Perturbation Analysis.

Tagaras and Cohen (1992) factor lead time into the model by considering two general locations with base-stock policies. Inventories transhipped at the end of each period take minimum time to arrive at their destinations, while new order arrivals are subject to certain lead time. The paper proposes four pooling options with respect to whether the transshipment should be triggered by threshold on-hand inventory levels and/or inventory positions. For complete pooling that maximizes profits of the two locations as a union, the paper also develops a heuristic solution which is shown to perform well under numerical simulations. Tagaras (1999) extends Tagaras and Cohen (1992) by considering three locations. Through simulation, it finds that risk-pooling benefit increases as the number of locations gets large and when locations face similar demand. However, the benefit is found to be less sensitive to transshipment policy.

[2]For the rest of the paper, we will use "identical" to describe locations or retailers with identical cost structure but possibly non-identical demand distributions.

Horizontal Relationships	Analysis of **Vertical Relationship** No	Yes
Centralized		
Physical Transshipment	Krishnan and Rao (1965); Tagaras (1989); Robinson (1990); Tagaras and Cohen (1992); Archibald et al. (1997); Herer and Rashit (1999); Tagaras (1999); Herer and Tzur (2001); Axsäter (2003ab); Herer and Tzur (2003) Wee and Dada (2005); Herer et al. (2006); Archibald (2007); Hu et al. (2008); Lee (1987); Axsäter (1990); Dada (1992); Alfredsson and Verrijdt (1999);	Dong and Rudi (2004); Zhang (2005); Grahovac and Chakravarty (2001);
Virtual Transshipment	McGillivray and Silver (1978); Parlar and Goyal (1983); Rajaram and Tang (2001); Netessine and Rudi (2003); Yang and Schrage (2008); Nagarajan and Rajagopalan (2008)	
Decentralized-Competitive		
Physical Transshipment	Anupindi et al. (2001); Rudi et al. (2001); Granot and Sošić (2003); Netessine and Rudi (2003); Hu et al. (2007); Zhao et al. (2005); Zhao et al. (2006); Zhao et al. (2008); Zhao and Atkins (2009); Huang and Sošić (2010b); Hezarkhani and Kubiak (2010); Comez et al. (2012);	Shao et al. (2011); Yan and Zhao (2011); Yan and Zhao (2012);
Virtual Transshipment	Parlar (1988); Wang and Parlar (1994); Lippman and McCardle (1997); Avşar and Baykal-Gürsoy (2002); Netessine and Rudi (2003); Zhao and Atkins (2009); Comez et al. (2012);	Anupindi and Bassok (1999); Jiang and Anupindi (2010);
Decentralized-Cooperative		
Physical Transshipment	Anupindi et al. (2001); Granot and Sošić (2003); Slikker et al. (2005); Sošić (2006); Huang and Sošić (2010a); Yan and Zhao (2012);	
Virtual Transshipment	Parlar (1988); Wang and Parlar (1994); Avşar and Baykal-Gürsoy (2002);	

Table 1: Literature on Inventory Transshipment

When there are non-trivial replenishment costs and possibly savings from joint replenishment, Herer and Rashit (1999) study a problem similar to Krishnan and Rao (1965) and Tagaras (1989) but for a single-period planning horizon. It discusses properties of optimal transshipment and replenishment policies under various cost structures.

For the situations where shortage cost is prohibitively high at one location, Axsäter (2003b) considers a model with unilateral transshipment. That is, transshipment is only allowed from the low-shortage-cost location to the high-shortage cost location. Through the analysis of a special network composed of three retailers, the paper develops approximations for key performance metrics including fill rates, expected on-hand stocks and expected shortages, hence reflecting the impact of unilateral transshipment on inventory systems.

In recent decades, a variety of periodical models arise to incorporate more factors that are worth further exploration. For finite planning horizon, Herer and Tzur (2001) analyze properties of optimal order and transhipment policies in a two-location system with dynamic deterministic demand. Efficient algorithm is developed in deriving the optimal policies. The results are extended to more than two locations by Herer and Tzur (2003).

To investigate the choice of transshipment policies, Wee and Dada (2005) study a network of one warehouse and n identical retailers applying base-stock policies. At the end of each period, unmet demand at each retailer can be fulfilled by leftover inventories at other retailers or the warehouse. The paper considers five systems with respect to transshipment policies: transhipment via retailers only, transhipment via the warehouse only, transshipment via both the retailers and the warehouse but priority is given to the retailer sites, transshipment via both the retailers and the warehouse but priority is given to the warehouse, and no transshipment. It identifies parameter regions under which a particular policy out performs others, as well as conditions that the warehouse should be open for transshipment.

For production subject to supply uncertainty, Hu et al. (2008) study a multi-period two-location problem in which the capacity at each site is random across the planning horizon. In each period, decisions have to be made on production quantity at each location, and inventory transshipment (rationing) after demand realization. The paper characterizes optimal state-dependent produce-up-to policy for production and floor-rationing policy for transshipment.

In studying vertical relationships in relevant distribution systems, Dong and Rudi (2004) consider a system with one manufacturer and n identical retailers. The model is under a single-period setup, so it generally applies to seasonal/perishable products or those with long lead times. The wholesale price between the manufacturer and the retailers can be exogenous or endogenous. For the endogenous setting, it finds that manufacturer's profit generally decreases with demand correlation between the retailers and increases with the number of retailers. In particular, the manufacturer generally benefits from transshipment among the retailers, while the retailers' profit can be less under transshipment when the manufacturer has the pricing power. While Dong and Rudi (2004) mainly use normal distribution to analyze the demands, Zhang (2005) further shows that the results hold for general demand distributions.

To allow more responsive transshipment, Archibald et al. (1997) study a periodic inventory systems with continuous demand (Poisson arrival). Different than all aforementioned papers, it considers the possibility that transshipment can happen in real time between two review epochs. Specifically in its two-location model, when one location runs out of stock, a request will be sent to the other location for possible inventory transshipment. The paper identifies optimal threshold conditions for the approval of such request, which will depend upon the inventory level at the second location, and time left to the next review epoch. Base-stock policy is also shown to be optimal in this model. Archibald (2007) extends these results to more than two locations, and proposes three transshipment heuristics that yield better performance than some commonly used ones.

Other than Archibald et al. (1997), most papers with continuous demand (e.g., Poisson arrival) adopt continuous inventory review. This line of research stems from determining optimal stock levels in part-repair systems, which usually consist of one central depot and several repair bases with limited service capacities or inventories (METRIC, Sherbrooke (1968)). Lee (1987) studies one of such systems in which M bases form into n disjoint groups. Each base in group i applies one-for-one replenishment policy with stocking level S_i. Within any group, bases are identical (including the same demand distribution) and can share inventories with each other. If a base runs out of stock when a customer arrives, a unit of inventory will be transhipped from other bases in the same group, or from the depot if none is available within the group. In case there are more than one bases available to transship from, the paper discusses several sourcing options: random pick, choose a base with maximum on-hand inventory, or with maximum on-hand inventory and minimum outstanding orders waiting at the depot. The paper derives approximations for expected backorders and units for transshipment, which can be used for finding optimal stocking levels.

Axsäter (1990) extends the model in Lee (1987) by considering non-identical locations. Using a different modeling technique, it further derives steady-state probability of the system and provides better approximates of service levels. Similar analysis is carried in Grahovac and Chakravarty (2001) but for a revised system. Grahovac and Chakravarty (2001) consider an extended supply chain consisting of a manufacturer, a distribution center, and general set of retailers. The lead time is stochastic from the manufacturer to the distribution centre, and deterministic from the distribution center to the retailers. Transshipment is triggered when the inventory level at one location is below a pre-determined emergency level. Different than its preceding works, the paper considers both the centralized setting (one decision maker determines initial stock levels and emergency triggers) and the vertically-decentralized setting (the distribution is the leader in determining stocking level and the retailers sets their stocking levels and triggers second). Through numerical experiments, it shows that transshipment may increase total cost and/or the total inventory, and that the stakeholders (the distributor and the retailers) may not always benefit from transshipment.

Axsäter (2003a) considers an (R, Q) replenishing policy for multiple locations facing independent Poisson demand. The decision involves whether all or part of the demand should be covered by transshipment. The paper develops an approximation to state-dependent expected cost, and derives transshipment rules accordingly. This leads to heuristic solutions on the reorder point and fixed batch quantity for each location.

While all the above papers assume that customers' demand can be backordered, Dada (1992) considers a problem in which demand could be lost if there is no inventory available in the system. The model involves one warehouse (with base stock level m) and N centres (with base stock 1). Demand of centre i follows Poisson distribution with rate λ_i. Each centre has a priority list of other centres that it will turn to in case of a stock out. If none of these centres has on-hand inventory, the demand will be satisfied by the warehouse if it has any on-hand or in-transit inventory. Otherwise, the demand will be considered lost, or equivalently, satisfied by costly sources outside the system. The paper first studies a one-warehouse-one-centre aggregated model, which is then used to approximate the performance of a one-warehouse-n-identical-centre system where the priority transshipment list is represented by a random transfer. Numerical test suggests that the approximation performs well when the results converge (e.g., when fill-rate is large in some examples). Alfredsson and Verrijdt (1999) study a similar problem but allowing a more general base stock level S_i at location i, and restricting transshipment from on-hand inventories only (Dada (1992) considers in-transit inventories also). Approximations for system performance measures are developed. There is no convergence issue with the results and the accuracy is supported by simulation.

2.2. Virtual transshipment in centralized models

The consideration of virtual transshipment arises with the advance of information technology, which allows customers to be more actively involved in the purchasing process — practically, searching for other in-stock locations when the one they attend is out of stock. If all customers are willing to search and travel, then effective management under centralized models can be achieved via a central warehouse as discussed in Eppen (1979). However, because searching and travel are usually costly, it is then natural that not all customers will engage in these activities. Complete pooling of inventories is then more often used as a bench mark of supply chain performance (e.g., see Anupindi and Bassok (1999) for a comparison between complete pooling and decentralized system with customer search). Although for centralized models there are less papers discussing virtual transshipment in a direct manner, a number of models on *partial substitutable products* can readily be adapted to virtual transshipment. *Partially substitution* implies that each product can be used to substitute for all other products with certain substitution probability (Rajaram and Tang (2001)). In the framework of virtual transshipment, one can interpret n partially substitutable products with acceptance probability α_{ij} (customers demanding product i will take product j as substitute) as n locations/retailers where, if location i stocks out, a fraction α_{ij} of its customers will buy at location j instead.

McGillivray and Silver (1978) study a periodic review system with two products of identical cost structure. Demand of either product can be partially transferred to the other if there is a stockout. The paper discusses the impact of this fraction on the inventory and the profitability of the system. A heuristic is developed to determine the optimal base-stock level. For a single-period problem with two partially-subsitituable products of general cost structures, Parlar and Goyal (1984) confirm the concavity of the profit function and provide necessary conditions in approximating the optimal inventory solution.

Rajaram and Tang (2001) investigate a single-period model with two partially-subsitituable products of identical cost-structure, whose demand can be correlated. The paper develops an approximation for the service rate. The heuristic is then applied to numerical studies in exploring the impact of the degree of substitution (or degree of search), demand uncertainty and demand correlation on inventory decision and consequently the profit.

Netessine and Rudi (2003) study a general model with n partially-substitutable products. They characterize structural results for the optimal ordering policy and show that centralized profit decreases with demand correlation. Yang and Schrage (2008) consider a similar setting and show that overall inventory can increase as a result of pooling, providing several realistic conditions that support this finding.

For two partially-substitutable products with identical cost structure, Nagarajan and Rajagopalan (2008) study a problem with fixed total demand (hence individual demands are negatively-correlated). The paper shows that state-independent base-stock policy is optimal for the two-product single-period problems as well as for many two-product multi-period problems. The policy may also be optimal for some multi-product single-period problems. A heuristic is derived from the base-stock policy, which is shown to perform well under low degree of substitutability (or low degree of search), low demand uncertainty, and high service level.

3. Decentralized Models

The study of decentralized systems soared as practitioners and researchers became more aware of supply chain relationship management under globalization. Particularly for the distribution system in Figure 1, decentralization implies that each retailer is now independently owned and operated. These then give rise to several distinctions from the centralized models in §2:

First, operational decisions and measures need to be revisited. The change in system structure results in inventory decisions that are now at the discretion of each retailer instead of by a central planner, the objectives that are now maximizing local profits instead of the total profit of all retailers, and the search for optimal solutions (or approximates) replaced by identifying equilibrium decisions (if there is any). One critical measure of decentralized systems under the control of multiple decision makers, is then the efficiency level compared with centralized systems.

Second, more dynamics arise on issues involving two or more retailers. One example is the allocation of cost and revenue generated by transshipment, which matters little in the centralized models but may affect retailers' incentive to share inventory in decentralized settings. Therefore, contract/mechanism design plays an important role in the study of decentralized models.

In addition, certain decentralized systems also allow for cooperative decision making among the retailers, which may greatly enhance the flow of goods and information across the supply chain.

In this section, we will first examine decentralized models in which retailers make competitive decisions (§3.1). Papers allowing retailers to form coalitions and make cooperative decisions will be discussed in §3.2. Finally, models involving both competitive and cooperative decisions (possibly in different stages of the game) are discussed in both subsections but with different focus. Within each subsection, we segment papers according to the nature of transshipment, i.e., physical or virtual. Papers studying both kinds of transshipment will be discussed only once, where it is most relevant.

3.1. Horizontal Relationships: Competitive Models

For independent retailers selling identical products at distinct yet adjacent geographic locations, competition exists in many aspects such as inventory levels and retail prices. This section will review papers in which relationships among retailers are purely competitive. That is, each retailer makes its operational decisions by properly taking into account others' decisions and reactions.

Physical Transshipment in Dencentralized-Competitive Models

We first summarize papers that model physical movements of excess inventories. Anupindi et al. (2001) establish a general distribution framework with n retailers. The retailers first make competitive decisions on order quantities, and then after demand realization, cooperative decisions on the pattern of transshipment. Obviously, how to allocate the profit generated out of transshipment will impact retailers' competitive ordering decisions. The paper considers several options of profit allocation. It finds that the dual solutions (by solving the linear programming problem of transshipment) cannot induce a first-best order decision. Certain fractional allocations can lead to a first-best order decision but lack other merits, and thus cannot coordinate the system either. Finally, the paper provides a contract in the form of fractional allocation with side payments (relevant to dual allocations) that coordinates the system.

While Anupindi et al. (2001) implicitly assume that retailers will share all their residual inventory or demand, Granot and Sošić (2003) extend this framework by allowing retailers to determine how much residuals to share. Specifically for dual allocations, it finds that retailers have incentives not to fully share their residuals, yielding total profit loss. However, for those rules that would not reduce the total profit of the system, such as Shapley value and fractional allocations, first-best order decision cannot be guaranteed. In respect to the coordinating contact provided in Anupindi et al. (2001), this paper also proposes an allocation rule that would not decrease the maximum profit of the system and would induce first-best order decision at the same time.

The dual allocations extensively studied in Anupindi et al. (2001) and Granot and Sošić (2003) represent the marginal value a unit of excess inventory or unfulfilled demand generates via transshipment. Therefore, the excess inventory or demand a retailer contributes to transshipment may be of different "value" depending on the demand realization. Thus this allocation rule, although agreed among the retailers ahead of time, determines prices of residual inventory and demand in an *ex post* manner.

Another widely used allocation rule sets the residual inventory or demand prices *ex ante*, i.e., before demand is realized. Rudi et al. (2001) consider a two-retailer decentralized system. It assumes that for each unit transhipped from location i to j, retailer i will charge retailer j a transshipment price c_{ij} and pay for the transshipment cost τ_{ij} herself. On the other hand, the profit p_j goes to retailer j who pays for the transshipment price c_{ij}. The paper studies how the equilibrium order decisions compare with the centralized order decisions. It also proposes a pair of transshipment prices c_{ij} and c_{ji} that coordinates the system. The problem is further investigated by Hu et al. (2007) who show that coordinating transshipment prices do not always exist between two general retailers. The paper discusses how its existence can be affected by cost and demand parameters.

In comparing allocation rules with different timing (*ex ante* vs *ex post*), Huang and Sošić (2010b) analyze equilibrium decisions and system's profit under both dual allocations and transshipment prices. It finds that the transshipment prices are more efficient with retailers of similar cost structure, and that dual allocations yield higher system profit when the retailers are more asymmetric in costs. In terms of practicability though, transshipment prices are relatively simple to understand and execute. However, unlike dual allocations, their applicability for more than two retailers were ambiguous. On this dimension, the paper identifies a one-to-one link between expected dual prices and coordinating transshipment prices, which is then used to develop heuristic transshipment prices for n-identical retailers. The paper also proposes heuristic transhipment prices for n general retailers by introducing one neutral central depot.

Other than the aforementioned allocations (dual prices and transshipment prices), Hezarkhani and Kubiak (2010) develop an implicit pricing mechanism that coordinates the general two-retailer decentralized system via General Nash Bargaining Solution. By this mechanism, the transshipment prices will depend upon retailers' order decisions.

While all the above papers consider single-period models, Zhao et al. (2005) study a two-retailer decentralized system under an infinite planning horizon. The demand is modelled as Poisson process, and inventory subject to continuous review. It is assumed that each retailer adopts an $(S,\ K)$ policy, where S represents the base-stock level and K the inventory rationing level. That is, if the on-hand inventory of retailer i is below K_i, he will not fulfill transshipment request from retailer j. The paper derives steady-state distribution of the inventory levels at each retailer, based on which closed-form cost functions are obtained. Two kinds of game are considered with regard to base-stock level decision as well as inventory rationing decision. In the first game, retailers make competitive decisions on base-stock levels (S_i, S_j) given the maximum amount of inventory one is willing to share with the other $(S_i - K_i, S_j - K_j)$. In the second game, base-stock levels (S_i, S_j) are given, and the retailers will have to determine inventory rationing levels (K_i, K_j) competitively. Existence of equilibrium is proved for both games.

Zhao et al. (2006) further study a network of n retailers. On top of the base-stock level S and inventory rationing level K as in Zhao et al. (2005), it also allows retailer to make a transshipment request once its inventory level falls below a certain level Z. The paper proves the optimality of the (S, K, Z) policy and discusses how to compute the parameters. Equilibrium solutions and the impact of the transshipment requesting level are explicitly discussed in a two-retailer network.

While transhipment is most often used to extend sales, Zhao et al. (2008) also examine its role in adding production flexibility. The paper studies a two-location production system where each location is modelled as a single-server make-to-stock queueing system. Transshipment can happen both prior to demand realization (hence a location can possibly produce for the other) as well as after demand realization (in fulfilling unsatisfied demand). The paper analyzes structure properties of the optimal production and demand filling policies, and develops heuristics that approximate optimal policies.

When the retailers may not have full knowledge of each other's demand, Yan and Zhao (2011) study a single-period model with one manufacturer and two retailers. The manufacturer determines wholesale price before retailers place their orders. The paper first analyzes a system with full information, and characterizes the impact of demand distribution on the optimal wholesale price. Results in this scenario imply that effective decision making requires demand variations only. In other words, lack of information on expected demand would not affect decision making. When demand information is private to the relevant retailer only, the paper discusses how Bayesian Nash equilibrium can be achieved. Finally, the paper proposes a mechanism that coordinates the system under full information, and yields very little efficiency loss under private information. The mechanism is similar to a two-part tariff plus transshipment payments. Yan and Zhao (2012) confirm the robustness of these results with more than two retailers.

Shao et al. (2011) investigate a single-period problem with two identical retailers and one manufacturer. The manufacturer sets wholesale price according to whether the retailers will engage in transshipment and the transshipment prices if they do. The paper finds that manufacturer in general prefers high transshipment prices between the retailers while retailers prefer the opposite. This leads to the manufacturer preferring dealing with centralized than decentralized retailers.

For continuous demand, Comez et al. (2012) consider a similar problem to Archibald et al. (1997) but in a single-period decentralized setting, and allowing both physical and virtual transshipment. It studies a one-manufacturer-two-retailer system, and assumes there is a single order opportunity for each retailer at the beginning of the season. Demand then arrives in Poisson process. For each arrival at a stock-out retailer, a (physical) transshipment request will be sent to the other retailer. The second retailer may either accept or reject the request. In case of a rejection, there is a nontrivial probability the customer will travel (virtual transshipment) and buy from the second retailer himself. The paper develops optimal holdback policy for the retailers regarding whether a (physical) transshipment request shall be accepted. The value and sensitivity of this policy are illustrated via numerical experiments. The paper also develops heuristic transshipment policy under Poisson demand when there are more than two retailers.

Virtual Transshipment in Dencentralized-Competitive Models

We next summarize papers concerning virtual transshipment, which has also been referred to as customer search, demand spillover, and partial substitution in certain contexts.

Parlar (1988) considers two decision makers selling substitutable products that have only one-period shelf life time. Each player deals with a random market of its own. Since the products are substitutable, a fraction a of player 1's customers will switch to player 2 if player 1 is sold out. A similar fraction b applies when player 2 stocks out. Each player has to determine the best order quantity with respect to the other's decision, which maximizes its one-period expected profit. The paper proves the existence and uniqueness of the Nash solution. A similar problem with three decision makers is studied by Wang and Parlar (1994), who suggest that substitution may increase inventory levels at the retailers.

Lippman and McCardle (1997) study a single-period problem with two competing firms of identical cost structure. Aggregated industry demand follows some known distribution, and customers will be allocated to each firm by a pre-specified rule — the paper considers four kinds of initial demand allocation rules that range from deterministic splitting to random splitting. In case of a stockout at firm i, a portion of the unfulfilled customers α_j will switch and attempt to buy from firm j instead. Existence of equilibrium order quantity is discussed, as well as how the initial demand allocation rule may affect the equilibrium. In particular, if all unsatisfied customers will visit the other firm ($a_i = a_j = 1$), then decentralization always yields higher stock levels than centralized systems. The analysis also extends to more than two retailers under some generic assumptions on demand.

Anupindi and Bassok (1999) study a supply chain with one manufacturer and two retailers. Inventories with the two retailers can be centralized or decentralized. In the decentralized system, retailers hold their inventories separately. A fraction of customers who find a retailer out of stock may "search" and turn to the second retailer for potential purchase. In the centralized case, retailers stock their inventories at one central warehouse, keeping only samples at local stores. Comparison of the two systems suggests that while retailers will always earn higher profit under centralization, the reverse may happen for the manufacturer and even for the entire supply chain if the degree of "search" is too high. The paper also discusses how endogenous wholesale prices/holding cost subsidy may impact retailers/manufacuturer's profits.

Netessine and Rudi (2003) also analyze decentralized retailers under partial substitution (which can be interpreted as virtual transshipment). It finds that while decentralization will always increase the inventories for one or more retailers (compared with the centralized case), it is also possible that some retailer may stock less due to decentralization. Specifically for identical retailers (in both cost and demand distribution), decentralization will increase the inventory level for every one.

Some recent papers compare the performance of physical vs. virtual transshipment in decentralized systems. Zhao and Atkins (2009) study a single-period game with two retailers selling partially substitutable products through deterministic price-sensitive markets. Regarding the unsatisfied demand, the retailers can either conduct physical transshipment at pre-specific transshipment prices, or let a fraction of the customers spillover to the in-stock retailer (virtual transshipment), who retains all the revenue. Performance of the system is compared under these two transhipment options. The key insight is that physical transshipment outperforms virtual transshipment when transshipment prices are high and competition is low.

Jiang and Anupindi (2010) study a similar problem on partially substitutable products, whereas the model considers a supply chain with one manufacturer and two identical retailers, and the markets are price-sensitive and uncertain. The paper models physical (resp. virtual) transshipment as retailer (resp. customer) -driven search. In either case, the search may not be complete, and its intensity is represented by the fraction of unsatisfied demand that will be covered (for physical transshipment) or spillover (for virtual transshipment). The paper considers several transhipment pricing schemes, including fixed price and prices proportional to that of the transshipment source or sink. It finds that a stakeholder would generally prefer physical transshipment over virtual transshipment if it is allowed to determine the transshipment prices. Otherwise, one might possibly be better off under virtual transshipment. Therefore, the preference for transshipment type often differs vertically across the supply chain, i.e., between the manufacturer and the retailers.

While most of the papers in this area focus on single-period decisions, Avşar and Baykal-Gürsoy (2002) analyze a customer-driven substitution problem under an infinite planning horizon. The results confirm that the Nash equilibrium strategy is in the form of stationary base-stock policy.

3.2. Horizontal Relationships: Cooperative Models

In recent years, a number of papers consider the possibility that retailers may form coalitions and make cooperative decisions that lead to win-win outcomes. The cooperative vision may greatly impact the retailers' ordering and sharing decisions. In addition, literature also analyzes equilibrium coalition structures among the retailers, especially whether the grand coalition would be stable. We next summarize papers in this area.

Physical Transshipment in Dencentralized-Cooperative Models

As discussed earlier in §3.1.1, Anupindi et al. (2001) establish a hybrid framework that allows retailers to make competitive order decisions and yet cooperate in the transshipment stage, assuming that all residual inventory or demand will be shared with each other. For a number of rules that allocate transshipment profit, the paper discusses whether they will be in the core — that is, if all retailers will form a single (grand) coalition for transshipment of residual inventories. For example, if residual inventory or demand is paid at their dual prices, then retailers would form grand coalition in transshipment.

However, under some fractional rules retailers may form small coalitions and transship inventories with only a limited number of others.

Granot and Sošić (2003) extend this framework by allowing retailers to determine how much to share as well as whom to share with. The paper discusses a variety of allocation rules by whether they may induce complete sharing or value-preserving sharing (in amounts that do not result in a decrease in the total additional profit), as well as if they are in the core (can induce grand coalition). It finds that although dual allocations are in the core, they cannot induce value-preserving sharing, and thus resulting with a loss in the total profit. On the other hand, value-preserving rules such as Shapley value and fractional allocations cannot induce grand coalition in the transshipment stage. These two kinds of allocations are furthered studied by Sošić (2006) and Huang and Sošić (2010a).

Sošić (2006) suggests that value-preserving rules such as Shapley value cannot induce grand coalition when retailers are myopic — that is, when retailers only consider immediate payoffs after coalition formation. For this reason, the paper allows the retailers to bear farsighted vision — to be concerned not only with their immediate payoffs but also with reactions of other retailers to their actions. It shows that grand coalition is stable among retailers generating identical transshipment revenues, and possibly when these are non-identical as well.

For dual allocations, Huang and Sošić (2010a) find that it could be a subgame perfect Nash equilibrium for retailers to share their residuals. The premises are that the game is repeated infinitely many times and the discount factor is large enough. For a large number of identical retailers, asymptotic behaviour for order quantity and threshold discount factor are analyzed. The paper also provides a contract that induces first best order decision if the residual profit is allocated by dual prices.

Slikker et al. (2005) consider a general n-newsvendor transshipment game in which cooperative decisions can be made on both order quantities and transshipment patterns. The paper shows that the core of the game is non-empty. In other words, no group of retailers has an incentive to split off from the grand coalition and form a smaller coalition.

In the context of information asymmetry, Yan and Zhao (2012) study an extended problem of Yan and Zhao (2011) that consists of more than two retailers who do not know others' demand distributions. The paper considers the possibility of both residual inventory sharing and information sharing, the latter of which is shown to be not obtainable in general. A mechanism is designed that can induce complete inventory sharing without information sharing. Its performance is shown to be quite close to first-best results under asymmetric information.

Virtual Transshipment in Dencentralized-Cooperative Models

For two substitutable products with customer search, Parlar (1988) analyzes both the equilibrium order quantities in the competitive setting and solutions in the cooperative setting. It shows that cooperative solutions generate higher total profit than Nash equilibrium solutions.

Under a similar model framework, Wang and Parlar (1994) consider three substitutable products with given customer search rates. Two or more players can form coalition to jointly determine order quantities and share excess stocks in avoiding penalties. Depending on whether side payments are allowed during profit allocation, the paper discusses conditions that lead to players forming grand coalition.

For an infinite planning horizon, Avşar and Baykal-Gürsoy (2002) also highlight the benefit of cooperation for a system of two substitutable products with customer search.

4. Conclusion and Future Directions

The study of inventory transshipment has been long lasting. The focus, methodologies, and solutions keep evolving with time, change of social structures, as well as technology development. This paper provides a brief overview of relevant works in transshipment that fits the general distribution network described in Figure 1. We acknowledge that some papers might have been left out of this review due to the limit of our focus. For the areas that have been examined, the classification in Table 1 suggests several issues that are understudied and may deserve future exploration:

Cooperation. The extensive use of cooperative game theory on transshipment problems only began in recent years. Given the limited amount of work in this area and the network nature of transshipment problems, there is a great potential to advance the understanding of opportunities and benefits from cooperation in distribution networks. While the current cooperative focus is mainly on transshipment partners and patterns, the effort can naturally be extended to other relevant activities including forecasting, ordering, inventory holding, customer relationship management and possibly pricing.

Vertical relationships. Table 1 partially reflects the lack of vertical-relationships study in transshipment literature. This is natural as transshipment initially arises as a network issue among parallel partners. However, vertical relations bring in externalities to the system, thus can significantly impact decision makings on the horizontal dimension. Therefore, identifying key issues on the vertical dimension is critical in fulfilling system-wise analysis. By Table 1, the greatest potential may lie in centralized virtual transhipment and decentralized cooperative transshipment. Manufacturers or suppliers may consider proper contract design that affects chain stores' order behaviour, or independent stores' alliance structure.

Virtual transshipment/Consumer-driven search. Due to its customer-driven nature, virtual transshipment was often modelled as a passive activity where customers shoulder the transshipment cost. The analysis of virtual transhipment was not put side-to-side with physical transshipment until the booming of information technology, which allows both the buyers and the sellers more flexibility between the two levers. To our best knowledge, not much analytical work has discussed the possibility of incorporating virtual transshipment in the operations strategy. This will involve influencing customers' search effort, as well as their preferences between actively searching versus passively accepting physical transshipment. These choices will impact upstream inventory decisions. Thus, the retailers should properly take them into account when determining customer relationship policies.

Pricing. Except for very few papers, transshipment literature generally considers exogenous market price. The scenarios where retailers have market power and are under the pressure for single-period decision making are understudied, yet prevalent in many industries like fashion, high tech, holiday toys, etc. An in-depth study of the power of pricing on inventory decisions, sharing decisions, transhipment decisions as well as supply chain vertical relationships, will be a valuable addition to the transshipment literature.

Empirical/Behaviorial studies. Existing literature in transshipment offers various policies, heuristics, and contracts that enhance the performance of the system. However, it is less known how these analytical findings and practical measures relate with each other. It would be of great value to identify key factors that prevent certain proposals from being adopted, and whether these factors could possibly be incorporated into existing models for more applicable solutions. In addition, understanding human behaviour on policy implementation can also help refining existing theoretical framework.

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