On modeling spot electricity markets. An overview

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Abstract

Liberalization of the electricity sector has opened a new field in economic modeling. Indeed, game theory and operation research techniques are implemented to model electricity firms’ competition. In particular, optimization models and time series analysis are useful to analyze allocation of resources, price forecast and many relevant issues in the study of electricity markets. In this paper we review actual trends of mathematical modeling applied to electricity markets.

Keywords: Electricity markets, Oligopoly models, Optimization, Time series analysis.

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1. Electricity markets and mathematical modeling

The electricity industry has experienced significant changes towards liberalization in order to achieve economic efficiency. In this new environment generating units no longer depend on state. Contrary to this, generation firms maximize their own profits. In most of OECD (Organization for Economic Co-operation and Development) countries firms compete by bidding price-quantity auctions in a centralized wholesale market (it is an e-marketplace) which set prices and quantities as a result of the interaction between supply and demand.¹ In addition, firms may sign long run contracts (called bilateral contracts) with large consumer units (industries or railway infrastructures, for instance), where price and quantities are committed in advance.

Electricity has two technical features that determine the complexity of any model aimed at explaining how this market works: The electricity production

¹In Spain, this virtual market is managed by OMEL, the Spanish market operator. More information about its organization and procedures, as well as electricity statistics, can be found at www.omel.es.
cannot be stored and its transport requires a physical network to reach final consumers.

For instance, the Spanish electricity market includes all the participants, operations, and rules that maintain the reliability of the network. Market architecture comprises the entire market, submarkets (generation, distribution and reselling), and the linkages between them. A wholesale spot market started operations in January 1998 when Act 54/1997 on the Spanish Power Sector came into force. This opened up the market to competition in generation and end-demand activities. Bilateral contracting was also introduced to allow long-term contracting to hedge against price volatility. After market clearance, OMEL (Operador del Mercado Eléctrico) dispatches the amount of electricity traded through REE (Red Eléctrica Española), the network infrastructure. Figure 1 describes the market organization and the linkages between participants under the new regime.

![Figure 1: Spanish electricity system.](image)

As a result of this new technical environment, electricity firms are exposed to significantly higher risks and they need for suitable decision models. Moreover, practitioners and regulatory agencies also need mathematical modeling for market design purposes but also for competition policy issues. Operation research and game theory play an important role in modeling electricity markets. The former provides elements for optimal allocation of resources as well as methods for weather forecast. The later is useful to model firms’ decisions in a context of imperfect competition. In addition, simulation techniques are implemented when the number of parameters are high and it is impossible to obtain explicit and qualitative results by using equilibrium models. In what follows, we review the main trends in this field.

Research developments in spot electricity markets can be divided in equilibrium and non-equilibrium models. Optimization techniques are used in equilib-
rium models that represent the overall market behaviour taking into consideration competition among all participants firms. Moreover, agent-based models are also useful in some environments. Non equilibrium models are an alternative way to equilibrium models when the problem under consideration is too complex to be addressed within a formal equilibrium framework. Their various purposes and scopes also imply distinctions related to market modeling, computational tractability and main uses.

2. Equilibrium models

Within this type of models we can distinguish agent-based models and optimization models. Models within the first group aim at representing explicitly the behaviour of each agent by using heuristic models of such behaviour. Market equilibrium is found under an appropriate methodology: The profit maximization program is approximated by a set of exogenous rules, which are specified for each market participant. The rationale for this approach is that complex non-cooperative games are difficult to solve, so that a heuristic description of agents’ behaviour may simplify the problem while being approximate enough. Hence, this approach determines the market price by means of market agents’ interaction rather than by the use of statistical description of the market. A representative example of agent-based simulation applied to power markets is Bower and Bunn (2000), where the simulation is designed to compare different market mechanisms. Generators engage in a bidding electricity market, and their behaviour is described by a learning algorithm driven by profit maximization and by the goal to reach a given utilization rate of their power plants.

In what follows we revise more in detail optimization models. Under this kind of models, we found single-firm (that is, monopolistic) optimization models and those models that use game theory (thus, taking into account strategic interaction of multiple firms). Both approaches use traditional mathematical optimization models. In addition, game theory approach provides solution concepts.

2.1. Single-firm models

Despite of equilibrium models consider in most of the cases a set of technical and economic constraints, single-firm models take the price clearing process as exogenous to the optimization program. Thus, the market revenue is a linear function in output. This approach is only suitable for almost perfect competition assumption as it neglects strategic interaction among firms. This type of models can be generally represented by

\[
\max_q \pi(q); \text{ s.t. } \phi(q) \leq \varphi
\]
where $q$ is the energy produced and $\pi$ is a vector of sources. $\pi(q)$ is the profit function and $\phi(q)$ is a vector of technology restrictions. An example of a deterministic model can be found in Gross and Finlay (1996) whereas a stochastic example which price uncertainty is Rajamaran et al (2001). A more extended version of a single-firm optimization models is to consider the price as a function of the firms’ output decisions. A leader in price model in a deterministic environment is found in Garcia et al (1999). Anderson and Philpott (2002) formulate this problem by constructing an optimal supply curve under uncertainty.

2.2. Multiple-firm models

In this subset we find those approaches which explicitly consider market equilibria by using traditional mathematical programming and game theory solution concepts. There are two main types of equilibrium models. The traditional Cournot competition where firms compete in quantity strategies and the most complex type based on supply function equilibria (SFE, hereinafter) where firms set multiple price-quantity pairs that draw a supply function. Under both approaches the basic equilibrium concept is Nash equilibrium: The market reaches equilibrium when each firm’s strategy is the best response to the strategies actually taken by its opponents.

Cournot equilibrium

Theoretical justification to apply Cournot equilibrium in electricity markets in the case of imperfect competition is suitable because easy of computability and also its straightforward intuitions. The mathematical structure of Cournot models turns out to be a set of algebraic equations, while the mathematical structure of SFE models turns out to be a set of differential equations. However, the use of Cournot models in electricity markets is reduced: It has frequently been used to support market power studies and also capacity decisions. A collection of essays regarding Cournot competition can be found in Daughety (1988). Borenstein et al. (1995) employed this theoretical market model to analyze Californian electricity market power instead of using the more traditional Hirschman–Herfindahl Index (HHI) and Lerner Index, which measure market shares and price-cost margins, respectively. Later on, Borenstein and Bushnell (1999) have extended this approach by developing an empirical simulation model that calculates the Cournot equilibrium iteratively.

Recently, Cournot approach is being used to model the issue of renewable sources and electricity spot markets. In Tamás et al (2010) and Zhou and Tamás (2010) the Cournot approach is used to test the effectiveness of feed in tariffs systems and green certificates to promote renewable sources in the electricity technology mix. In both papers, Cournot quantities are intended as capacities. Each firm maximize profits subject to the regulator qualification on a particular
share of renewables,

$$\max_{q_i} \left[ p_g(q_{ig}, q_{-ig}) + \varphi(q_{ig}) + p(q_i, q_{-i})q_i - C_{ig} - C_i - \alpha p_g q_{ig} \right]$$

s.t.: \( \sum_{i=1}^{n} q_{ig} \geq \alpha Q \).

where \( q_{ig} \) are amount of renewables, \( \alpha \) is the ratio of renewables over the total amount of energy, and \( \varphi \) is a function of either, a feed in tariff system, or the price of renewable sources. \( p_g \) is the price of renewables and \( Q \) is total quantity. \( C_{ig} \), and \( C_i \) are generic cost functions. The term \( \alpha p_g q_{ig} \) apply only in the case of any firm output is below the ratio \( \alpha \). Solving by Kuhn-Tucker techniques a unique solution is reached, under the assumption of strictly concavity of the objective function.

**Supply function equilibria**

Klemperer and Meyer (1989) showed that when a firm faces a range of possible residual demand curves, it expects, in general, a bigger profit expressing its decisions in terms of a supply function that indicates the price at which it offers different quantities to the market. This is the SFE approach developed by Klemperer and Meyer (1989). It has been proven to be an extremely attractive line of research for the analysis of equilibrium in wholesale electricity markets. Calculating an SFE requires solving a set of differential equations instead of the typical set of algebraic equations that arises in traditional equilibrium models. Hence, SFE models have thus considerable limitations concerning their numerical tractability. Research focused on these models concentrate on market power analysis and representation of electricity pricing by linearizing the SFE model. The possibility of obtaining reasonable medium-term price estimations with the SFE approach is considerably attractive. For numerical tractability reasons, researchers have recently focused on quasi-linear SFE models (in which demand is linear and marginal costs are linear or affine) where SFE can be obtained in terms of linear or affine supply functions. Green (1996) considers the case of an asymmetric n-firm oligopoly with linear marginal costs facing a linear demand curve whose slope remains invariable over time.\(^2\) An SFE expressed in terms of affine supply functions is obtained. Baldick et al. (2004) extend previous results to the case of affine marginal cost functions and capacity constraints. They assume that after the shock is realized, all markets clear: Each firm produces at the point on its supply function intersects the realized residual demand. In a world with exogenous uncertainty (an exogenous demand shock) a given firm has a set of profit-maximizing points—one for each realization of the uncertainty—when it knows its competitor’s pure strategy equilibrium behaviour. In this

\(^2\)In Green and Newbery (1992) and Green (1999) supply function equilibrium approach is also used to model electricity markets.
setting, a firm can generally achieve higher expected profits by committing to a supply function that by committing to a fixed price or a fixed quantity, because a supply function allows better adaptation to the uncertainty.

They assume that firms have identical cost functions $C(\cdot)$, with $C'(q) > 0$, for all $q > 0$, and $0 < C''(q) < \infty$ for all level of quantity $q \geq 0$. Without loss of generality, let $C'(\cdot) = 0$. If $C'(\cdot) = \alpha > 0$, then the analysis below can be applied to solve for supply functions expressed in terms of $\tilde{p} = p - \alpha$. Firms choose SF’s simultaneously, without knowledge of the realization of $\varepsilon$. After the realization of $\varepsilon$, SF’s are implemented by each firm producing at a point $(p^*(\varepsilon), S_k(p^*(\varepsilon)))$ such that $D(p^*(\varepsilon)) = S_i(p^*(\varepsilon)) + S_j(p^*(\varepsilon))$; That is, demand matches total supply, provided that a unique such price $S_k(p^*(\varepsilon))$ exists. A firm $i$ solves

$$\max_p p[D(p, \varepsilon) - S_i(p)] - C(D(p, \varepsilon) - S_i(p))$$

with first order condition,

$$D(p, \varepsilon) - S_i(p) + [p - C'(D(p, \varepsilon) - S_i(p))][D_p(p, \varepsilon) - S_i'(p)] = 0.$$

By assuming that $D_{p\varepsilon} = 0$ for all $(p, \varepsilon)$, demand is translated horizontally by the shock $\varepsilon$. Then, if we write $D_p(p, \varepsilon(S(p), p))$ simply as $D_p(p)$, from the first order condition above we get,

$$S'(p) = \frac{S(p)}{p - C'(S(p))} + D_p(p) \equiv f(p, S). \tag{2.1}$$

In any SFE for unbounded support of $\varepsilon$, the outcome corresponding to any given value of $\varepsilon$ is intermediate between the outcome that would arise if firms learned $\varepsilon$ and then competed in the Cournot fashion and the outcome that would arise if they observed $\varepsilon$ and then competed using Bertrand strategies.

*Klemperer and Meyer’s linear example*

For a market in which the demand and marginal cost curves are both globally linear, they propose this linear example in order to compute the solutions to (2.1) and show the existence of a unique SFE when $\varepsilon$ has full support. Industry demand is $D(p, \varepsilon) = \varepsilon - mp$, where $m > 0$ and $\varepsilon$ has (full) support $[0, \infty)$. Firms have identical quadratic cost curves $C(q) = \frac{c}{2}q^2$, where $c > 0$. The differential equation (2.1) becomes

$$S'(p) = \frac{S(p)}{p - c \cdot S(p)} - m,$$
Let the SF for each firm $S(p) = \beta p$. Then, after some algebra

$$S(p) = \beta p = \frac{1}{2}(-m + \sqrt{m^2 + \frac{4m}{c}})p.$$  

Ciarreta and Gutiérrez-Hita (2006) reach the same result for the linear specification in a game involving $n$ firms and extend results to the symmetric repeated game in order to analyze the scope for collusion sustainability. The slope of this SF, market prices and outputs are intermediate between the Cournot and Bertrand cases (in which firms choose quantities and prices, respectively, after seeing the realization of the uncertainty). It follows that firms’ profits are also intermediate between these cases. Turnbull (1983) and Robson (1981), have shown for the market of this example that the function $S(p) = \beta p$ is the unique SFE when firms are restricted to choosing linear supply functions. They also note that given the nature of uncertainty and its support, the SFE are independent of the distribution of the uncertainty, remaining unchanged even as the distribution becomes more and more sharply peaked at a specific point and approaches the no-uncertainty case. When a SFE under a natural kind of uncertainty is unique, it may be a natural candidate for the correct SFE in the limiting case of no uncertainty.

In Green (1996) a linear supply function model with asymmetric firms competing in supply functions as in Klemperer and Meyer (1989) is used to model the electricity spot market in England and Wales. Every time, generators submit prices for each generating set. As the industry is dominated by two multi-plant companies (this is also the case of the spanish electricity system) when a large company raises the bids of some of its stations, those stations will be used less, but the system marginal price will be higher, and so the firm’s remaining stations will earn more. A firm with enough power stations would profit by raising some of its bids above its cost. There are several studies that have provided rigorous backing for this claim. Bolle (1992) applied the supply function techniques derived by Klemperer and Meyer (1989) to the Pool, and showed that this theoretical model implied a SFE price well above marginal costs. In Bolle’s model there are $n$ generators, which compete by submitting a supply function $q_i(p) : \mathbb{R}^+ \rightarrow \mathbb{R}^+, i = 1, \ldots, n$ which state the amount they would be willing to produce $q_i$ at any price $p$. As Klemperer and Meyer (1989) it is assumed that demand is stochastic. The model assumes that the game takes place between the producing firms. On the spot market, the producers sell electricity at price $p$ to the transmission and distribution company (TDC, hereinafter). Aggregate demand is,

$$N = n - mq.$$  

where $m$ is constant and $n$ is a random variable with density function $f(n)$ on
Later on, households use electricity as input and pay $p = q$.

The suppliers evaluate the outcome by their expected profits,

$$G_i = \int_{\bar{n}}^{\underline{n}} [(p^*(n, q) - c_i) \cdot s_i(p^*(n,q)) \cdot f(n)] \, dn.$$

Bolle assumes that in the determination of $q$, the price $p$ is set equal to $p^*$, i.e. the step where the TDC choose $q$ is omitted. The customers have to pay the spot price. He finds a continuum of symmetric equilibria (where all players have the same profit). Despite of the TDC acts as a Walrasian auctioneer, consumers pay spot prices, i.e $p = q$. Bolle states that when demand $N(p,n)$ and costs $v_i(x)$ fulfill the following conditions, (i) $N_{pp} \leq 0$, and $N_{pn} = 0$, (ii) $n \in [0, \infty)$, (iii) $v_i(x) = v(x)$, (iv) $v''(x) \geq 0$; then,

- there is at least one equilibrium combination of SF’s;
- every equilibrium is symmetric and the SF’s are increasing;
- as long as $f(n) > 0$ for all $n$, the equilibria are independent of $f(n)$;
- for linear demand and quadratic cost functions there is a unique equilibrium; prices and quantities lie between Cournot and Bertrand solution;
- if there is a unique equilibrium then market prices decrease with an increasing number $k$ of producers.

The proof of this theorem can be found in Klemperer and Meyer (1989). The essential assumption which causes symmetry, uniqueness and neighbourhood to the Cournot and Bertrand solutions is (ii), i.e. potentially unbounded demand. Without this assumption there are asymmetric solutions and multiplicity of symmetric solutions. Thus, he investigates how the number of producers $k$, the spread of the distribution of demand parameters $\bar{n} - \underline{n}$, and the slope of the demand function $m$ influence the spot price. To do so, he modifies Klemperer and Meyer’s model in two ways. First, the lower bound of demand variation is assumed to be finite and, second, the model is simplified by the assumption of zero costs. In particular, if $\underline{n}$ is close to $\bar{n}$ (if the spread of the distribution of $n$ is too small) then the convergence to zero prices is slow. Indeed, in countries where electricity supply is large as in Germany, Spain or Great Britain, electricity companies try to keep the necessary capacity as small as possible in order to use as much base load as possible (they try to decrease the spread $\bar{n} - \underline{n}$). Bolle remarks that under the assumption of competition on an electricity market is in SF’s, one must conclude that this market may be governed by tacit collusion.

Challenges remain for the supply function equilibrium models when the firms are asymmetrical (having different cost functions) and have operating constraints

\[^3\text{This is the underline assumption in Klemperer and Meyer (1989).}\]
such as capacity limits. Anderson & Hu (2008) stress that the only case in which an asymmetric supply function equilibrium can be easily found is one in which the supply functions are linear (strictly affine). This form of SFE can be found whenever the cost functions are quadratic (linear marginal cost) and the demand is linear. However there may also be nonlinear solutions under these conditions, which will be difficult to find analytically. It is natural to consider piecewise linear supply function equilibria in which different price ranges have different linear solutions. This approach has been used successfully by Baldick et al (2004).

3. Non equilibrium models

In this group are those models devoted to the study of market prices without a detailed description of market players’ behaviour. It is worth to highlight that this does not mean that strategic behaviour is not represented, but that it is defined exogenously. They can be classified under two broad groups: econometrics and self-dispatch models. Econometrics models consider that markets are aggregately represented by the market price, and then carrying out a statistical analysis of such prices. Self-dispatch models consider the profit maximization problem of a single agent, assuming that rivals’ and demand’s behaviour is represented exogenously (typically, determined by means of statistical estimation).

3.1. Econometrics models

The econometrics approach is based on time series analysis applied to electricity spot prices. To do so, it considers that all relevant information about the market is contained in spot prices, so that the observation of such prices allows us to describe any market situation. Hence, a critical assumption underlying this modeling approach is that all information is contained in historical prices. By assuming that all possible situations have occurred before, it should be possible to characterize the price distribution for any future period (see for instance Weron (2008) for a specialization on electricity prices). Moreover, price forecasting must represent both the production cost and the strategic behaviour of market players. This task is specially difficult in power markets, because the price distribution resulting from the combination of both effects is considerably complex. This has motivated the use of explanatory variables to aid in the definition of price distributions. The idea is to use the regression of prices over other variables, which in principle are easier to model. This kind of approach is often referred to as fundamental models. For instance, Skantze et al. (2000) expressed spot prices as a function of load and supply, which were stochastic variables. Reserve margin as a fundamental driver has been used by Mount et al. (2006) and Anderson and Davison (2008). More recently, Coulon and Howison (2009), among others, use an heuristic definition of the bid curve to transform fundamen-
tal drivers into power prices. We describe here two econometric models based on time series analysis. It is assumed that price values are recorded at fixed time intervals. The presented models are selected based on an inspection of the main characteristics of the hourly price series. In competitive electricity markets this series presents the following features,

1. High frequency and nonconstant mean and variance.

2. High volatility and high percentage of unusual prices (mainly in periods of high demand).

3. Multiple seasonality and calendar effects (corresponding to a daily and weekly periodicity, and weekends and holidays).

Taking the relationship between prices and electricity demand, an initial approach to model electricity prices is the use of a linear regression model. However, this approach has a serious problem due to the presence of serial correlation in the error. Thus, the model is not appropriate. Moreover, forecasts has a very limited accuracy. Therefore, it is necessary to develop models that can handle correlated errors and also following a recursive scheme. An outline of this scheme is as follows,

- Step 1; A model is identified assuming certain hypotheses and an initial selection of parameters is chosen.
- Step 2; The model parameters are estimated.
- Step 3; If the hypotheses of the model are validated go to Step 4; otherwise go to Step 1 to refine the model.
- Step 4; The model can be used to forecast.

A generic dynamic regression approach

An usual technique which is proposed to overcome the serial correlation problem uses a dynamic regression model. In this model, the price at hour $t$ is related to the values of past prices at hours $t - 1, t - 2, ...$ and to the values of demands at hours $t, t - 1, t - 2, ...$. This is done to obtain a model that has uncorrelated errors. In Step 1, the selected model used to explain the price at hour $t$ is,

$$p_t = c + \omega^d(B)d_t + \omega^p(B)p_t + \varepsilon_t$$

where $p_t$ is the price at time $t$, $c$ is a constant, and $d_t$ is the electricity demand at time $t$. Functions $\omega^d(B) = \sum_{l=0}^{K} \omega^d_l B^l$ and $\omega^p(B) = \sum_{l=1}^{K} \omega^p_l B^l$ are polynomial functions of the backshift operator $B$ such that $B^l d_t = d_{t-l}$, and $B^l p_t = p_{t-l}$.
where \( l = 1, 2, \ldots, k \) is the number of delays. \( \omega^d(B) \) and \( \omega^p(B) \) depend on parameters \( \omega^d_l \) and \( \omega^p_l \), respectively, whose values are going to be estimated under step 2. Finally, \( \epsilon_t \) is an error term. In Step 1, this term is assumed to be a series drawn randomly from a normal distribution with zero mean and constant variance \( \sigma^2 \) (a white noise process). The efficiency of this approach depends on the election at step 1 of an appropriate parameters in \( \omega^d(B) \) and \( \omega^p(B) \) to achieve an uncorrelated set of errors. This selection have to be done in steps 1–3.

**Transfer function approaches**

A second set of methods to avoid serial correlation are those that include a serially correlated error. Such an approach is called transfer function model. It is assumed that the price and demand series are both stationary (i.e., with constant mean and variance). The general form proposed to model the (price, demand) transfer function is

\[
p_t = c + \omega^d(B)d_t + N_t
\]

where \( p_t \) is the price at time \( t \), \( c \) is a constant, \( d_t \) is the electricity demand at time \( t \), and \( \omega^d(B) = \sum_{l=1}^{K} \omega^d_l B^l \) is a polynomial function of the backshift operator defined as in the previous approach. \( N_t \) is a disturbance term that follows an ARMA model,

\[
N_t = \frac{\theta(B)}{\phi(B)} \epsilon_t
\]

with \( \theta(B) = 1 - \sum_{l=1}^{\Theta} \theta_l B^l \) and \( \phi(B) = 1 - \sum_{l=1}^{\Phi} \phi_l B^l \), where \( \Theta \) and \( \Phi \) are the maximum number of delays in \( \theta(B) \) and \( \phi(B) \), respectively. Both of which are polynomial functions of the backshift operator. Finally, \( \epsilon_t \) is an error term. Such a model relates actual prices to demands through function and actual prices to past prices through function \( \omega^d(B) \) and actual prices to past prices through function \( \phi(B) \). Following a recursive scheme (similarly to the dynamic regression approach), the parameters different from zero in \( \omega^d(B) \), \( \theta(B) \), and \( \phi(B) \) are selected. Final selected parameters different from zero for function \( \omega^d(B) \) are the same as those selected for the dynamic regression model.

**3.2. Self-dispatch approaches**

An important stream of engineering literature has focused on the description of output decisions of power producers. The problem typically involves a certain agent owning a generation portfolio, who has to decide on the operation of her power plants. When this decision-making process takes place in a market environment, the market price plays a central role in dispatch decisions. Thus, this can be thought of as a method to describe power prices, which obtains them as the solution of the decision-making process of a single producer. Broadly
speaking, the problem consists in the optimization of firms’ decisions considering the market response as input data. To motivate the logic for this modeling approach, consider a certain agent that has to decide on the output of each power plant $g_j$ of her generation portfolio at each point in time $t$. Let me use the following program to represent the profit-maximization problem of a market player,

$$\begin{align*}
\max_{g_t^j} & \sum_{t,j} \pi_t g_t^j - C_t^j(g_t^j) \\
g_t^j & \leq g_t^{\text{MAX}} \\
R_t(g_t^j) & = 0
\end{align*}$$

where $C_t^j(g_t^j)$ is a cost function with $\partial C_t^j(g_t^j)/\partial g_t^j > 0$ and $\partial^2 C_t^j(g_t^j)/\partial (g_t^j)^2 \geq 0$. $g_t^{\text{MAX}}$ is a set of maximum output constraints, and $R_t(g_t^j)$ represent some additional constraints concerning technology, network requirements and other physical restrictions. This program represents that optimal production decisions are those that maximizes the firm’s profits. It is assumed that technical characteristics are known, so the central issue of the above problem is the definition of the market response, in this case the market price. Several models proposed in the literature have assumed that the firm solving this self-dispatch problem is too small to affect market prices (price takers). Hence, given the market price, the problem consists in calculating the optimal output of each power plant. However, this kind of model is of limited use as a price model, because the price description is a pre-requisite for solving the model. Nonetheless, other proposals have developed methods to represent more complex market interactions. The idea is to consider an exogenously defined function $\pi_t(g_t)$, where $g_t = \sum_t g_t^j$ is the aggregate output of the firm. The methodology builds on the idea that it is possible to represent competitors’ behaviour by means of their output decisions for each possible market price. This modeling approach has been analyzed in depth in García-González et al. (2000) and Anderson and Philpott (2002). On the other hand, residual demands are typically step functions, because of among other factors, maximum output constraints of power plants. To avoid the statistical estimation of steps related to maximum output, Pereira et al. (2005) proposes a two-level optimization, where the upper level consists in maximizing agents profits and the lower level clears the market. The resulting problem, thus, gives the residual demand in terms of the lower level constraints. In addition, Vázquez and Vázquez (2009) suggest constructing agents’ offers using dual information from the linear relaxation of the previous problem (which is justified using the optimal price results of Vázquez (2003)).

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References


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